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ON THE STABILITY OF PERIODIC POINCARÉ SOLUTIONS OF HAMILTONIAN SYSTEMS IN THE DEGENERATE CASE*

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The sufficient conditions for the orbital stability of periodic Poincaré solutions in the case of natural degeneracy are obtained for a certain class of Hamiltonian systems with two degrees of freedom. The orbital stability of periodic Poincaré solutions in the problem of periodic motions relative to the centre of mass of a dynamically symmetric artificial satellite with an inertia ellipsoid close to a sphere, in a circular orbit, is investigated as an application.

1. Poincaré's theorem. The autonomous system with two degrees of freedom being examined has a Hamiltonian of the form

$$H = H_0(G_1) + \omega G_2 + \sum_{k=1}^{\infty} \varepsilon^k H_k(g_1, g_2, G_1, G_2) \tag{1.1}$$

Here g_1, g_2 are generalized coordinates, G_1, G_2 are the corresponding generalized momenta, ω is a constant quantity, and ε is a small parameter. It is assumed that H is a 2π-periodic function of the generalized coordinates, analytic in all its arguments in some domain of phase space $M = \mathbf{T}^2 \times Q\ (Q \subset \mathbf{R}^2)$. When $\varepsilon = 0$ the equations of motion with Hamiltonian (1.1) have the solution

$$G_1 = x_1, \quad G_2 = x_2, \quad g_1 = \omega_1 t + y_1, \quad g_2 = \omega t + y_2, \quad \omega_1 = dH_0(x_1)/dx_1 \tag{1.2}$$

where $x_i, y_i\ (i = 1, 2)$ are constant quantities. When $\varepsilon = 0$ the Hessian of H with respect to the variables G_1, G_2 equals zero identically and we have the degenerate case. We assume that the hypotheses of Poincaré's theorem on the existence of periodic solutions of a system with Hamiltonian (1.1) are satisfied, i.e., initial values $x_i, y_i\ (i = 1, 2)$ exist /1/ such that the following conditions are satisfied:

1) the generating solution (1.2) is periodic with period T, i.e., $\omega_1 T$ and ωT are multiples of 2π, $\omega = l\omega_1/m\ (l \in \mathbf{Z},\ m \in \mathbf{N})$

2) $$\left.\frac{d^2 H_0(G_1)}{dG_1^2}\right|_{G_1 = x_1} \neq 0$$

3) $$\frac{\partial \langle H_1 \rangle}{\partial x_2} = \frac{\partial \langle H_1 \rangle}{\partial y_2} = 0$$

4) $$\det \begin{Vmatrix} \dfrac{\partial^2 \langle H_1 \rangle}{\partial y_2^2} & \dfrac{\partial^2 \langle H_1 \rangle}{\partial y_2 \partial x_2} \\ \dfrac{\partial^2 \langle H_1 \rangle}{\partial x_2 \partial y_2} & \dfrac{\partial^2 \langle H_1 \rangle}{\partial x_2^2} \end{Vmatrix} \neq 0$$

$$\left(\langle H_1 \rangle = \frac{1}{T} \int_0^T H_1(\omega_1 t + y_1, \omega t + y_2, x_1, x_2)\, dt\right)$$

We choose the initial instant such that $y_1 = 0$ for any ε and for $t = 0$. If the initial values of $x_i, y_i\ (i = 1, 2)$ have been chosen such that conditions 1-4 are satisfied, then the equations of motion with Hamiltonian (1.1) have, for sufficiently small $\varepsilon \neq 0$, a periodic solution of period T, which can be expanded in a series in powers of the small parameter ε with periodic coefficients of period T and which reverts to solution (1.2) when $\varepsilon = 0$. We will write this solution as

$$\begin{aligned} &g_1 = \omega_1 t + \varepsilon g_1^{(1)}(\omega_1 t) + \ldots, \quad g_2 = \varkappa \omega_1 t + y_2 + \varepsilon g_2^{(1)}(\omega_1 t) + \ldots \\ &G_1 = x_1 + \varepsilon G_1^{(1)}(\omega_1 t) + \ldots, \quad G_2 = x_2 + \varepsilon G_2^{(1)}(\omega_1 t) + \ldots; \quad \varkappa = l/m \end{aligned} \tag{1.3}$$

All functions on the right-hand sides of (1.3) are periodic with period $2\pi m$ in the variable $w_1 = \omega_1 t$; the ellipses denote terms of higher order of smallness in ε.

2. Introduction of perturbations in a neighbourhood of the periodic solution. We will investigate the orbital stability of the periodic solution (1.3). For this

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*Prikl.Matem.Mekhan., 47, 5, 728-736, 1983 we change to the new canonical variables w_1, q_2, I_1, p_2 $(w_1 = \omega_1 t)$ such that we obtain the periodic solution (1.3) when $q_2 = p_2 = I_1 \equiv 0$. The variables q_2, p_2, I_1 are perturbations of the periodic solution (1.3), where q_2, p_2 are perturbations of the first order of smallness and I_1, as an action variable, is a quantity of the second order of smallness.

We introduce the perturbations by the formulas

$$g_1 = w_1 + \sum_{k=1}^{\infty} \varepsilon^k g_1^{(k)}(w_1) \tag{2.1}$$

$$g_2 = \varkappa w_1 + y_2 + \sum_{k=1}^{\infty} \varepsilon^k g_2^{(k)}(w_1) + q_2$$

$$G_1 = x_1 + \sum_{k=1}^{\infty} \varepsilon^k G_1^{(k)}(w_1) + I_1 - \varkappa p_2 + \sum_{k=1}^{\infty} \varepsilon^k G_*^{(k)}(w_1, q_2, I_1, p_2)$$

$$G_2 = x_2 + \sum_{k=1}^{\infty} \varepsilon^k G_2^{(k)}(w_1) + p_2$$

The functions $G_*^{(k)}$ are selected such that transformation (2.1) is canonical and $G_*^{(k)}(\omega_1, 0, 0, 0) \equiv 0$ for any $k = 1, 2, \ldots$. For the generating function

$$S = \sum_{k=0}^{\infty} \varepsilon^k S_k(g_1, g_2, I_1, p_2) \tag{2.2}$$

we have the equations

$$\frac{\partial S}{\partial I_1} = w_1, \quad \frac{\partial S}{\partial p_2} = q_2, \quad \frac{\partial S}{\partial g_1} = G_1, \quad \frac{\partial S}{\partial g_2} = G_2 \tag{2.3}$$

From (2.1)-(2.3) we find

$$S_0 = g_1(x_1 + I_1 - \varkappa p_2) + g_2(x_2 + p_2) - y_2 p_2 \tag{2.4}$$

$$S_1 = -(I_1 - \varkappa p_2) g_1^{(1)}(g_1) - p_2 g_2^{(1)}(g_1) + g_2 G_2^{(1)}(g_1) + \int_0^{g_1} \left\{ G_1^{(1)}(\xi) - (\varkappa \xi + y_2) \frac{dG_2^{(1)}(\xi)}{d\xi} \right\} d\xi$$

$$G_*^{(1)} = -(I_1 - \varkappa p_2) \frac{dg_1^{(1)}(w_1)}{dw_1} - p_2 \frac{dg_2^{(1)}(w_1)}{dw_1} + q_2 \frac{dG_2^{(1)}(w_1)}{dw_1}$$

3. To find the stability conditions for the periodic solutions (1.3) we will make use of Barrar's theorem /2/. The Hamiltonian of the perturbed motion, expanded in powers of $I_1, p_2, q_2, \varepsilon$ in a neighbourhood of the initial values from which the solutions (1.3) arise, with due regard to the fact that in the solutions (1.3) the energy integral equals

$$H_0(x_1) + \omega x_2 + \varepsilon \{H_1(w_1, \varkappa w_1 + y_2, x_1, x_2) + \omega_1 G_1^{(1)}(w_1) + \omega G_2^{(1)}(w_1)\} + O(\varepsilon^2)$$

is written as

$$H^* = \omega_1 I_1 + \frac{1}{2} \varkappa^2 \frac{d^2 H_0}{dx_1^2} p_2^2 - \frac{1}{6} \varkappa^3 \frac{d^3 H_0}{dx_1^3} p_2^3 - \varkappa \frac{d^2 H_0}{dx_1^2} I_1 p_2 + \tag{3.1}$$
$$\frac{1}{2} \frac{d^2 H_0}{dx_1^2} I_1^2 + \frac{1}{24} \varkappa^4 \frac{d^4 H_0}{dx_1^4} p_2^4 + \frac{1}{2} \varkappa^2 \frac{d^3 H_0}{dx_1^3} I_1 p_2^2 +$$
$$\varepsilon \left\{ \frac{1}{2} D^2 H_1 + \frac{1}{6} D^3 H_1 + I_1 D \frac{\partial H_1}{\partial x_1} + \frac{1}{24} D^4 H_1 + \frac{1}{2} I_1 D^2 \frac{\partial H_1}{\partial x_1} + \right.$$
$$\left. \frac{1}{2} \frac{\partial^2 H_1}{\partial x_1^2} I_1^2 + G_1^{(1)}(w_1) L_1 + G_*^{(1)} \left(\frac{d^2 H_0}{dx_1^2} (I_1 - \varkappa p_2) + L_2 \right) \right\} +$$
$$R(w_1, q_2, I_1, p_2, \varepsilon)$$

$$L_1 = L_2 + \frac{1}{2} \frac{d^3 H_0}{dx_1^3} I_1^2 + \frac{1}{2} \varkappa^2 \frac{d^4 H_0}{dx_1^4} I_1 p_2^2$$

$$L_2 = \frac{1}{2} \varkappa^2 \frac{d^3 H_0}{dx_1^3} p_2^2 - \frac{1}{6} \varkappa^3 \frac{d^4 H_0}{dx_1^4} p_2^3 - \varkappa \frac{d^3 H_0}{dx_1^3} I_1 p_2$$

$$D = q_2 \frac{\partial}{\partial y_2} + p_2 \frac{\partial}{\partial x_2} - \varkappa p_2 \frac{\partial}{\partial x_1}$$

(R relative to ε has an order of smallness higher than the first, relative to q_2 and p_2, higher than the fourth, and relative to I_1, higher than the second).

The Hamiltonian of the perturbed motion is a $(2\pi m)$-periodic function of the variable w_1. We represent it in the form

$$H^* = \Phi_1 + \varepsilon \Phi_2$$

$$\Phi_1 = \frac{1}{2\pi m} \int_0^{2\pi m} H^*(w_1, q_2, I_1, p_2, \varepsilon) \, dw_1, \quad \langle \Phi_2 \rangle = \frac{1}{2\pi m} \int_0^{2\pi m} \Phi_2 \, dw_1 \equiv 0$$

Let us consider a Hamiltonian K, which is the quadratic part of Φ_1 in the variables q_2, p_2

$$K = ap^2 + \varepsilon cqp + \varepsilon bq^2. \tag{3.2}$$

$$a = \frac{1}{2}\varkappa^2 \frac{d^2H_0}{dx_1^2} + \frac{1}{2}\varepsilon\left\{\frac{\partial^2\langle H_1\rangle}{\partial x_2^2} + \varkappa^2\frac{\partial^2\langle H_1\rangle}{\partial x_1^2} - 2\varkappa\frac{\partial^2\langle H_1\rangle}{\partial x_1\partial x_2} + \frac{1}{2}\langle G_1^{(1)}\rangle\varkappa^2\frac{d^3H_0}{dx_1^3}\right\}, \quad \langle G_1^{(1)}\rangle = -\frac{\partial\langle H_1\rangle}{\partial x_1}\left(\frac{d^2H_0}{dx_1^2}\right)^{-1} \tag{3.3}$$

$$b = \frac{1}{2}\frac{\partial^2\langle H_1\rangle}{\partial y_2^2}, \quad c = \frac{\partial^2\langle H_1\rangle}{\partial x_2\partial y_2} - \varkappa\frac{\partial^2\langle H_1\rangle}{\partial x_1\partial y_2}$$

If

$$z = 4\varepsilon ab - \varepsilon^2c^2 > 0$$

then the characteristic equation corresponding to (3.2) has two purely imaginary complex-conjugate roots $\pm i\sqrt{z} \equiv \pm i\sqrt{\varepsilon}\Omega$. Otherwise, the periodic solution (1.3) is unstable.

As a result of the canonical transformation

$$\begin{gathered} w_1 = w_1', \quad q_2 = \varepsilon^{1/4}\alpha\sqrt{2I_2'}\sin w_2' - \varepsilon^{1/4}\beta\sqrt{2I_2'}\cos w_2' \\ I_1 = \varepsilon I_1', \quad p_2 = \varepsilon^{1/4}\alpha^{-1}\sqrt{2I_2'}\cos w_2' \\ (\alpha = \mathbf{sign}\, b\,(\Omega\,(2\mid b\mid)^{-1})^{1/2}, \quad \beta = c\,(2\mid b\mid\Omega)^{-1/2}) \end{gathered} \tag{3.4}$$

having the valence $1 \mid \varepsilon$ we obtain a new Hamiltonian of the perturbed motion

$$\begin{gathered} H^{**} = \omega_1 I_1 + \sqrt{\varepsilon}\{K_1(I_1, I_2) + K_2(w_1, w_2, I_1, I_2)\} \\ \left(K_1 = \frac{1}{2\pi m}\int_0^{2\pi m}\Phi_1(w_2, I_1, I_2)\,dw_2, \ \frac{1}{(2\pi m)^2}\int_0^{2\pi m}\int_0^{2\pi m}K_2\,dw_1\,dw_2 \equiv 0\right) \end{gathered} \tag{3.5}$$

In (3.5) we have omitted the primes on the new variables. Transformation (3.4) enables us to examine the variation of the variable I_2 in the ring $V_2 = \{\rho_1 \leqslant I_2 \leqslant \rho_2;\ \rho_1, \rho_2 > 0\}$ and the Hamiltonian H^{**} is an analytic function in all its arguments in a domain of phase space $M^* = \mathbf{T}^2 \times V$, where $V = V_1 \times V_2$, $V_1 \subset \mathbf{R}^1$; V_1, V_2 are closed sets.

We will write out the expression for $K_1(I_1, I_2)$ to within terms of the first order of smallness in ε and of the second order of smallness in I_1, I_2

$$\begin{gathered} K_1(I_1, I_2) = \Omega\,\mathrm{sign}\,b I_2 + \frac{1}{2}\sqrt{\varepsilon}\frac{d^2H_0}{dx_1^2}I_1^2 + \sqrt{\varepsilon}\frac{\alpha^4}{16}\frac{\partial^4\langle H_1\rangle}{\partial y_2^4}I_2^2 + \\ \varepsilon\frac{1}{2}\left(\varkappa^2\alpha^{-2}\frac{d^3H_0}{dx_1^3} + \alpha^2\frac{\partial^3\langle H_1\rangle}{\partial y_2^2\partial x_1}\right)I_1I_2 + O(\varepsilon^{3/2}, I_i^2I_j); \quad i, j = 1, 2 \end{gathered}$$

We consider the determinant

$$\det\begin{Vmatrix} \dfrac{\partial^2(\omega_1I_1 + \sqrt{\varepsilon}K_1)}{\partial I_i\partial I_j} & \dfrac{\partial(\omega_1I_1 + \sqrt{\varepsilon}K_1)}{\partial I_i} \\ \dfrac{\partial(\omega_1I_1 + \sqrt{\varepsilon}K_1)}{\partial I_j} & 0 \end{Vmatrix} = \sqrt{\varepsilon}N; \quad i, j = 1, 2$$

$$N = -\sqrt{\varepsilon}\,\omega_1^2\frac{\alpha^4}{16}\frac{\partial^4\langle H_1\rangle}{\partial y_2^4} + O(\varepsilon^{3/2}, I_j); \quad j = 1, 2$$

If $N \not\equiv 0$, then the Hamiltonian H^{**} of the perturbed motion, defined in (3.5), satisfies all the hypotheses of Barrar's theorem and, consequently, the periodic solution (1.3) is orbitally stable. When $\omega = 0$ all the arguments remain true, but here it is necessary formally to set $l = 0$ in all the calculations. Thus, we have proved the following:

Theorem. Let the Hamiltonian of an autonomous system with two degrees of freedom be defined by Eq. (1.1) and let the initial values of the generating solution (1.2) be chosen such that conditions 1-4 of Poincaré's theorem on the existence of periodic solutions of the perturbed system are satisfied. Then, if these initial values satisfy the conditions

$$\begin{gathered} \frac{d^2H_0}{dx_1^2}\frac{\partial^2\langle H_1\rangle}{\partial y_2^2} > 0, \quad \omega \neq 0 \\ \frac{\partial^2\langle H_1\rangle}{\partial x_2^2}\frac{\partial^2\langle H_1\rangle}{\partial y_2^2} - \left(\frac{\partial^2\langle H_1\rangle}{\partial x_2\partial y_2}\right)^2 > 0, \quad \omega = 0 \end{gathered} \tag{3.6}$$

$$\begin{gathered} \frac{\partial^4\langle H_1\rangle}{\partial y_2^4} \not\equiv 0, \quad \omega \neq 0 \\ \left[\alpha^2\frac{\partial^2}{\partial y_2^2} + \left(-\beta\frac{\partial}{\partial y_2} + \alpha^{-1}\frac{\partial}{\partial x_2}\right)^2\right]^2\langle H_1\rangle \not\equiv 0, \quad \omega = 0 \end{gathered} \tag{3.7}$$

then the periodic solutions (1.3) are orbitally stable.

4. Periodic motions of an artificial satellite. As an application we consider the problem of the periodic motions of a dynamically symmetric rigid body relative to the centre of mass in a circular orbit in a central gravitational field.

At the present time steady-state and planar periodic motions relative to the centre of mass of a dynamically symmetric artificial satellite in a circular orbit have been most completely studied. In a number of papers $(2\pi m)$-periodic solutions have been constructed in an elliptic orbit, coinciding when $e=0$ with $(2\pi m/n)$-periodic Liapunov solutions in the neighbourhood of the steady-state solution, which yield planar motions, and their stability in the linear approximation has been investigated (m and n are simple prime integers). A survey of the methods mentioned is given in /3/ *). The problem of periodic motions relative to the centre of mass of a dynamically symmetric artificial satellite with an energy ellipsoid close to a sphere in a circular orbit has been examined in /4/. Here we refine certain results of /4/ connected with the proof of the existence of periodic Poincaré solutions and we make a strictly non-linear analysis of the orbital stability of the resultant solutions.

We fix an arbitrary point on the satellite's orbit, taking it to be the orbit's perigee. In Fig.1, $OXYZ$ is a König coordinate system with origin at the satellite's centre of mass. Its axis OY is directed along the binormal to the orbit, while OX and OZ are directed, respectively, along the transversal and the normal to the orbit at its perigee. The angles ρ and σ determine the orientation of the kinetic moment vector $\mathbf{L}$ relative to the coordinate system $OXYZ$, ϑ is the angle between the vector $\mathbf{L}$ and the satellite's axis of dynamic symmetry (the Oz axis), ψ is the satellite's angle of rotation around the vector $\mathbf{L}$, φ is the angle of rotation of the vector $\mathbf{L}$ around the satellite's axis of dynamic symmetry.

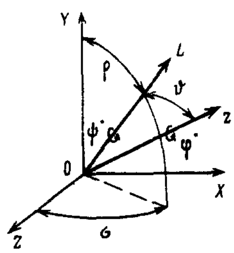

Fig.1

The satellite's motion relative to the centre of mass in a central gravitational field can be described by a system of canonical equations with the Hamiltonian /5/

$$H = K(L, l) - U(L, L_n, \psi, \sigma - \nu), \quad \nu = \omega_0 t$$
$$L_n = L\cos\rho \; (|L_n| \leqslant L), \quad l = L\cos\vartheta \; (|l| < L)$$

Here ω_0 is the angular velocity of the motion of the satellite's centre of mass along the orbit, L is the modulus of the kinetic moment vector, and L_n, l are the projections of the kinetic moment vector onto the normal to the orbit's plane and onto the axis of dynamic symmetry, respectively. The kinetic energy K and the force function U, respectively, equal

$$K = \frac{1}{2}\{(L^2 - l^2)/A + l^2/C\}, \quad U = \frac{3}{2}\omega_0^2(A - C)\gamma^2$$
$$\gamma = \beta\sqrt{1-\alpha^2}\cos S - \frac{1}{2}\sqrt{1-\beta^2}(1-\alpha)\sin(\psi + S) + \frac{1}{2}\sqrt{1-\beta^2}(1+\alpha)\sin(\psi - S)$$
$$S = \nu - \sigma, \quad \alpha = \cos\rho = L_n/L, \; \beta = \cos\vartheta = l/L$$

Here A and C are, respectively, the satellite's equatorial and polar moments of inertia ($A \neq C$), and γ is the cosine of the angle between the radius-vector of the satellite's centre of mass relative to the centre of attraction and the axis of dynamic symmetry. Since the angle φ is a cyclical coordinate, its corresponding momentum l is an integral of the motion $l = l_0$ and the order of the equations of motion can be reduced by two. The transformation $\lambda = \sigma - \nu$ leads to an autonomous system with two degrees of freedom and with the Hamiltonian

$$H^* = H - \omega_0 L_n = K(L, l_0) - \omega_0 L_n - U(L, L_n, \psi, \lambda) \tag{4.1}$$

We shall seek periodic solutions of the canonical equations of motion with Hamiltonian (4.1) when the satellite's inertia ellipsoid is close to a sphere. Let

$$A = J_0 + \varepsilon A_1, \; C = J_0 + \varepsilon C_1, \quad \varepsilon \ll 1$$

We expand the Hamiltonian (4.1) in series in powers of the small parameter ε up to terms of the first order of smallness in ε, inclusive

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*) See also: SIDORIUK M.E., Certain problems of the motion of artificial satellites relative to the centre of mass under the action of a gravitational moment. Dissertation for the degree of Candidate of Physico-Mathematical Sciences. Moscow, MFTI, 1981.

$$H^* = \frac{1}{2} L^2/J_0 - \omega_0 L_n + \varepsilon H_1 + O(\varepsilon^2), \tag{4.2}$$

$$H_1 = -A_1 L^2/2J_0^2 - \frac{3}{2}\omega_0^2 (A_1 - C_1)\gamma^2$$

When $\varepsilon = 0$ in the generating motion

$$L = L_0, \quad L_n = L_{n_0}, \quad \psi = \omega t + \psi_0, \quad \lambda = -\omega_0 t + \lambda_0, \quad \omega = L_0 / J_0 \tag{4.3}$$

the kinetic moment vector executes a translation along the orbit with angular velocity ω_0, while the satellite rotates around this vector with constant angular velocity ω.

We assume that the generating solution is periodic with period T. If conditions 1-4 of Poincaré's theorem (Section 1) are satisfied, then for fairly small $\varepsilon \neq 0$ periodic solutions of period T exist, arising from (4.3). From the equations of motion with Hamiltonian (4.1) it follows that periodic Poincaré solutions are possible in a circular orbit in only two cases: a) $\omega = \omega_0$, b) $\omega = 2\omega_0$. Conditions 1,2 are satisfied; we will investigate the remaining conditions in each case.

Let $\omega = \omega_0$, i.e., $L_0^* = J_0\omega_0$. In this case the function $\langle H_1 \rangle$ defined by (4.2) and by the [illegible]lity within the parentheses [illegible] condition 4 equals

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otherwise, it is unstable.

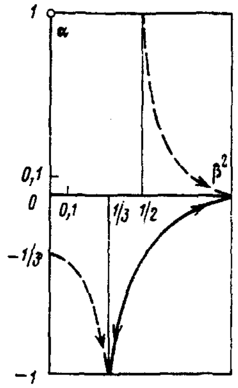

Fig.2

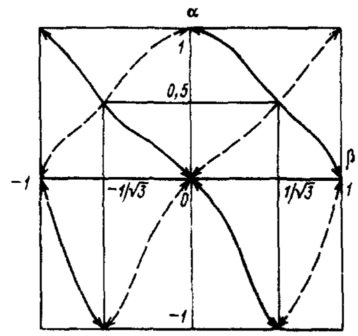

Fig.3

Now let $\lambda_0 = \pi/2$, $\lambda_0 = 3\pi/2$, then

$$\alpha = \frac{1-\beta^2}{7\beta^2-3}, \quad \beta^2 \ldots$$

$\{x: -1 < x < -1/\sqrt{3},\ 0 < x < 1/\sqrt{3}\}$ there correspond two distinct initial values of $\alpha = L_{n_0}/L_0$, while for solutions of the second type, to each $\beta \in B_4 = \{x: -1/\sqrt{3} < x < 0,\ 1/\sqrt{3} < x < 1\}$ there also correspond two distinct values of α. In /4/ the initial values for the angles λ which correspond to periodic Poincaré solutions are incorrect.

The stability conditions for the solutions are written as

$$\frac{d^2H_0}{dL_0^2}\frac{\partial^2\langle H_1\rangle}{\partial\lambda_0^2} = 3\frac{\omega_0^2}{J_0}(A_1 - C_1)\beta(1+\alpha)\sqrt{1-\beta^2}\times\ \sqrt{1-\alpha^2}\sin 2\lambda_0 > 0 \qquad (4.9)$$

$$\frac{\partial^4\langle H_1\rangle}{\partial\lambda_0^4} = -12\omega_0^2(A_1 - C_1)\beta(1+\alpha)\sqrt{1-\beta^2}\sqrt{1-\alpha^2}\sin 2\lambda_0 \neq 0$$

The second of these conditions is satisfied if $\alpha, \beta \neq 0, \pm 1$, while the first condition is equivalent to

$$(A_1 - C_1)\,\beta\sin 2\lambda_0 > 0 \qquad (4.10)$$

From (4.10) it follows that solutions of the first type are orbitally stable if $A_1 > C_1$ and $0 < \beta < 1$, i.e., $-\pi/2 < \vartheta < \pi/2$, or if $A_1 < C_1$ and $-1 < \beta < 0$, i.e., $\pi/2 < \vartheta < 3\pi/2$. Periodic solutions of the second type are orbitally stable if $A_1 > C_1$ and $-1 < \beta < 0$ or if $A_1 < C_1$ and $0 < \beta < 1$. Hence we see that if periodic solutions of the first type are orbitally stable, then solutions of the second type are unstable, and vice versa.

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ON THE IMPULSIVE MOTION OF A RIGID BODY AFTER IMPACT WITH A ROUGH SURFACE*

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An absolutely rigid plane body in contact with a plane surface of finite area, at each point of which the friction is locally defined by Coulomb's law, with a constant sliding coefficient of friction, is considered. A more precise model of the motion of a body over a rough surface /1/ is obtained. Differential equations of a plane rigid body (a plate) with a circular contact area are derived. The relation between the sliding velocity of the centre of the base area and the angular velocity of the plate is obtained in special cases. The condition under which the instantaneous centre of the base velocity in the course of impulsive motion coincides identically with the base area centre is derived.

The collision between a rigid and a rough surface has been investigated under conditions of point contact (/2/ etc.)

1. Let us consider the basic assumptions made in /1/ on the interaction between a rigid body with a plane base and a plane rough surface, when the body moves on it.

For absolutely rigid bodies and planes the problem is indeterminate, since contact occurs at an infinite number of points. Hence, a small deformation of the surface proportional to

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